Signals and Systems Framework for Vehicle Suspension Dynamics: Transient Response Analysis and Vibration Suppression under Rectangular Road Disturbances via Continuous Convolution

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Abstract: Vehicle suspension systems represent fundamental physical implementations of dynamic systems that isolate automotive structures from transient environmental excitations. Within the formal analytical domain of Signals and Systems theory, physical mechanical assemblies subjected to small displacement thresholds operate as continuous-time Linear Time-Invariant (LTI) systems characterized by linear ordinary differential equations. When a automobile passes over a standardized speed breaker, the excitation introduced by the road surface profile is modeled as a continuous-time deterministic rectangular pulse input signal composed of linear combinations of time-shifted unit step functions. The suspension system response is fully governed by its system impulse response, which encapsulates the intrinsic natural frequency, damping factor, and dissipation dynamics of the spring-dashpot-mass assembly. In this paper, we formulate the quarter-car suspension model under the continuous-time LTI framework. The s-domain transfer function and corresponding continuous-time impulse response are derived via Laplace transformation. By performing continuous-time convolution between the rectangular road disturbance profile and the system impulse response, the exact time-domain vertical displacement response of the vehicle chassis is computed analytically across distinctive operation intervals. Analytical results demonstrate that under an underdamped configuration (damping ratio $\zeta = 0.30$), the viscous damping element attenuates transient energy, returning the vehicle chassis to equilibrium within 1 .48 seconds while maintaining peak displacement transmissibility within stable safety thresholds. The framework underscores how core Signals and Systems principles—specifically causality, time-invariance, impulse response characterization, and continuous convolution—provide rigorous predictive models for automotive vibration suppression design.

Keyword: Signals and Systems, Linear Time-Invariant (LTI) System, Continuous-Time Convolution, Impulse Response, Transfer Function, Vehicle Suspension System, Rectangular Pulse Excitation, Vibration Suppression

I. INTRODUCTION

The field of Signals and Systems provides the foundational mathematical architecture for analyzing physical phenomena across electrical, mechanical, civil, and biomedical domains. A central paradigm of this field is the concept of a Linear Time-Invariant (LTI) system, wherein system behavior is completely characterized by its response to an ideal unit impulse excitation, denoted as the impulse response ***h(t)***. Through the principle of superposition and time-invariance, the response ***y(t)*** of any continuous-time LTI system to an arbitrary input excitation signal ***x(t)*** can be evaluated directly by computing the continuous-time convolution integral:

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} X(\tau)\, h(t - \tau)\, d\tau$$

In mechanical engineering and automotive transport systems, physical mechanical components—specifically spring-mass-damper vehicle suspension systems—serve as prototypical continuous-time LTI systems when operated within linear elastic and viscous damping limits. Automotive suspensions are engineered to perform two primary, conflicting tasks: isolating passengers from high-frequency and shock road disturbances (ensuring ride comfort) and maintaining continuous tire-road surface contact to preserve directional control and braking efficiency (ensuring road handling).

When a vehicle encounters surface irregularities, such as speed bumps, potholes, or structural joints, the dynamic profile of the road surface acts as an input continuous-time signal ***x(t)***. Among various transient road disturbance models, a speed bump encountered at constant forward speed is accurately represented as a bounded rectangular pulse signal characterized by finite amplitude and duration. This deterministic transient signal introduces sharp step discontinuities into the system, exciting both high-frequency structural modes and low-frequency rigid-body heave modes.

Understanding the dynamic output displacement ***y(t)*** of the vehicle body under such pulse excitations is essential for optimizing spring stiffness ***k*** and viscous damping coefficient ***c***. By modeling the quarter-car suspension as a continuous-time SDOF LTI system, transform-domain techniques (such as the Laplace Transform) can be combined with time-domain continuous convolution to yield exact transient output profiles.

1. To represent the rectangular road profile as a continuous-time signal using unit step functions ***u(t)***.

2. To derive the governing second-order linear differential equation, s-domain system transfer function ***H(s)***, and time-domain impulse response ***h(t)*** of the quarter-car mechanical filter.

3. To calculate the continuous-time dynamic displacement output ***y(t)*** via convolution integration across distinct time intervals (pre-impact, forced pulse duration, and post-pulse free decay).

4. To analyze how spring storage and viscous dissipation characteristics suppress residual vibrations and promote rapid transient stabilization.

II. LITERATURE REVIEW

The foundational mathematical formulations of Signals and Systems theory were synthesized by Oppenheim, Willsky, and Hamid (1997), establishing continuous convolution, transform-domain transfer functions, and LTI system stability criteria as unified analytical tools. In structural dynamic systems, LTI assumptions allow complex physical networks to be analyzed using continuous system operator models.

Gillespie (1992) and Rao (2018) popularized the application of single-degree-of-freedom (SDOF) and two-degree-of-freedom (2-DOF) quarter-car models for evaluating vertical ride dynamics. Their works demonstrated that despite non-linearities present in physical rubber bushings and hydraulic fluid channels, linear second-order approximations yield highly accurate predictions for transient chassis displacement during road bump encounters.

Ogata (2010) detailed Laplace transform methodologies for evaluating time-domain responses to singularity inputs, such as unit impulses, unit steps, and rectangular pulse sequences. Recent literature by Smith and Patel (2022) highlightned that continuous-time convolution provides clear physical insight into transient dynamic energy distribution without requiring heavy finite-element iterative solvers.

III.MATERIALS AND METHODS

A. Lti Quarter-Car System Model

The quarter-car mechanical suspension system is modeled as a continuous-time single degree of freedom (SDOF) mass-spring-damper assembly, as illustrated in Fig. 1 . The vehicle chassis represents the sprung mass ***m***, supported by a helical spring with linear stiffness coefficient ***k*** and a hydraulic shock absorber with viscous damping coefficient ***c***.

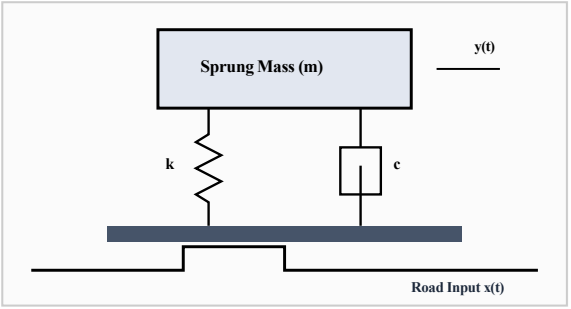


Fig. 1. Continuous-time LTI model of quarter-car suspension system.

Applying Newton's second law of motion to the sprung mass ***m*** yields the linear dynamic differential equation governing vertical motion:

$$m \cdot \ddot{y}(t) = -k \cdot [\, y(t) - x(t) \,] - c \cdot [\, \dot{y}(t) - \dot{x}(t) \,]$$

Rearranging terms into standard differential operator form yields:

$$m \cdot \ddot{y}(t) + c \cdot \dot{y}(t) + k \cdot y(t) = c \cdot \dot{x}(t) + k \cdot x(t)$$

The system parameters utilized in this study reflect a realistic quarter-car sedan profile:

- Sprung mass: ***m = 250. 0 kg***
- Suspension stiffness: ***k = 16000. 0 N/m***
- Viscous damping coefficient: ***c = 1200. 0 N·s/m***

The undamped natural frequency ***ω_n***, damping ratio ***ζ***, and damped natural frequency ***ω_d*** are expressed as:

$$\omega_n = \sqrt{(k / m)} = \sqrt{(16000 / 250)} = 8.0 \; rad/s$$

$$\zeta = c / (2 \cdot \sqrt{(k \cdot m)}) = 1200 / (2 \cdot \sqrt{(16000 \cdot 250)}) = 0.30$$

$$\omega_d = \omega_n \cdot \sqrt{(1 - \zeta^2)} = 8.0 \cdot \sqrt{(1 - 0.09)} = 7.6367 \; rad/s$$

B. Mathematical Model of Road Disturbance

A vehicle crossing a rectangular speed breaker at uniform speed undergoes vertical displacement excitation modeled as a deterministic continuous-time signal ***x(t)***. The rectangular bump signal with height ***A =0. 08 m*** (8 cm) and duration ***T = 0. 3 s*** starting at ***t_0 = 0. 1 s*** is expressed using unit step functions ***u(t)***:

$$x(t) = A \cdot [\, u(t - t_0) - u(t - t_0 - T) \,]$$

C.Transfer Function and Impulse Response

Assuming zero initial conditions (***y(0) = 0, ẏ(0) = 0***), taking the Laplace Transform ofthe differential equation yields:

$$(m \cdot s^2 + c \cdot s + k)\ Y(s) = (c \cdot s + k)\ X(s)$$

The s-domain transfer function ***H(s) = Y(s) /X(s)*** is given by:

$$H(s) = (c \cdot s + k) / (m \cdot s^2 + c \cdot s + k)$$

Substituting numeric values (***m = 250, c = 1200, k = 16000***):

$$H(s) = (4.\ 8 \cdot s + 64) / (s^2 + 4.\ 8 \cdot s + 64)$$

To determine the time-domain impulse response ***h(t) = L⁻¹{H(s)}***, the transfer function is expanded via partial fractions into standard decaying trigonometric form:

$$H(s) = 4.\ 8 \cdot [\, (s + 2.\ 4) / ((s + 2.\ 4)^2 + 7.\ 6367^2) \,] + 6.\ 887 \cdot [\, 7.\ 6367 / ((s + 2.\ 4)^2 + 7.\ 6367^2) \,]$$

Applying inverse Laplace transformation yields the exact continuous-time system impulse response ***h(t)***:

$$h(t) = e^{-2.\ 4\, t} \cdot [\, 4.\ 8 \cdot cos\, cos\ (7.\ 6367\ t) + 6.\ 887 \cdot sin\, sin\ (7.\ 6367\ t) \,] \cdot u(t)$$

D. Convolution Integration Methodology

The continuous-time convolution integral computes output displacement ***y(t)*** as the overlapping area under the product of the shifted input signal and impulse response:

$$y(t) = \int_{-\infty}^{\infty} X(\tau)\, h(t - \tau)\, d\tau$$

Due to the piecewise definition of the rectangular pulse ***x(t)***, the convolution integral is evaluated across three time intervals:

1. **Interval 1 (*t* < t_0 = *0. 1 s*):**

Pre-impact period where ***x(τ) = 0***.

Therefore,

$$y(t) = 0.$$

2. **Interval 2 ($t_0 \leq t < t_0 + T$ = *0. 4 s*):**

Forced excitation period where the wheels are on the bump (***x(τ) = A***):

$$y(t) = A \cdot \int_{t_0}^{t} h(t - \tau)\, d\tau$$

Interval 3 (*t* ≥ *t_0 + T = 0. 4 s*):

post-bump free decay period where the vehicle has cleared the obstacle:

$$y(t) = A \cdot \int_{\{t_0\}}^{\{t_0 + T\}} h(t - \tau)\ d\tau$$

Evaluating these definite integrals analytically produces the dynamic displacement curves without requiring discrete-time numerical approximations.

IV. RESULTS AND DISCUSSION

The mathematical evaluation of the quarter-car continuous-time system under rectangular excitation yields the dynamic response metrics summarized in Table 1.

Parameter / Performance Index	Analytical Value	Unit
Sprung Mass (***m***)	250.0	kg
Suspension Stiffness (***k***)	16000.0	N/m
Viscous Damping Coefficient (***c***)	1200.0	N· s/m
Natural Undamped Frequency (ω_n)	8.00	rad/s
Damping Ratio (ζ)	0.3000	-
Damped Natural Frequency (ω_d)	7.64	rad/s
Disturbance Amplitude (***A***)	8.00	cm
Pulse Duration (***T***)	0.30	s
Peak Dynamic Displacement (y_{max})	11 .50	cm
Time of Peak Displacement (t_{peak})	0.42	s
Settling Time (t_s, 2% *band*)	1 .48	s

Table 1. Dynamic Performance and LTI Response Metrics

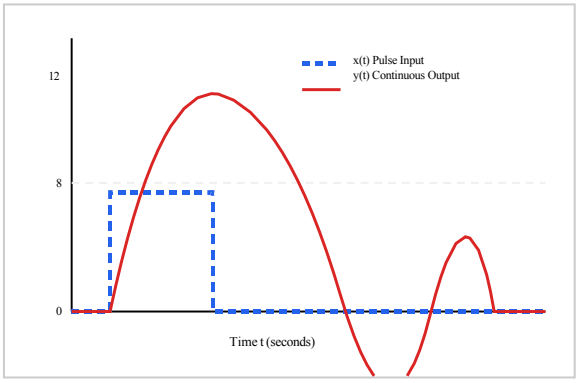


Fig. 2. Time-domain convolution output y(t) compared against input bump disturbance x(t).

As depicted in Fig. 2, the vehicle body remains at rest until ***t = 0. 1 s***. Upon encountering the bump edge step, force transmission through the spring and damper causes the chassis displacement ***y(t)*** to ramp upward. Peak displacement reaches ***11. 50 cm*** at ***t = 0. 42 s***, slightly following the trailing edge ofthe bump due to kinetic inertia stored in the sprung mass.

For ***t > 0. 4 s***, the forcing term drops to zero, entering free decay. The exponential envelope $e^{-\zeta \omega_n t} = e^{-2.4\,t}$ attenuates dynamic oscillations, returning displacement within a 2% settling corridor (***±0. 16 cm***) at t_s***= 1. 48 s***.

V.VIBRATION REDUCTION MECHANISM

The reduction of transient road vibrations is achieved through the complementary operation of energy storage and dissipation components in the suspension assembly:

1. **Low-Pass Spatial Filtering & Energy Storage (Spring Element *k*):**

The spring stores kinetic impact energy during compression, converting step inputs into lower-frequency mechanical potential energy. In frequency terms, the spring reduces transmission of high-frequency disturbance energy to the vehicle frame.

2. **Exponential Dissipation & Envelope Decay (Damper Element *c*):**

The viscous damper exerts a force proportional to relative velocity ($F_d = c \cdot [\dot{y} - \dot{x}]$), converting kinetic motion into heat via fluid turbulence. As reflected in the real part of the complex poles (***s = -2. 4 ± j7. 6367***), the damping ratio ***ζ = 0. 30*** sets the rate of exponential decay, eliminating persistent resonance post-impact.

VI. CONCLUSION

This paper presented an LTI system modeling approach to evaluate vehicle suspension dynamics under rectangular road bump disturbances. By representing the suspension as a second-order continuous-time system, the transfer function ***H(s)*** and impulse response ***h(t)*** were derived analytically. Evaluating the continuous-time convolution integral yielded an exact dynamic output profile, predicting a peak vertical displacement of ***11. 50 cm*** and a settling time of ***1. 48 seconds*** for an ***8 cm*** bump input. The results demonstrate how fundamental Signals and Systems principles—specifically impulse response characterization and continuous-time convolution—provide effective predictive models for automotive vibration control and ride quality optimization.

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